Universal energy distribution for interfaces in a random-field environment

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We study the energy distribution function $\rho(E)$ for interfaces in a random-field environment at zero temperature by summing the leading terms in the perturbation expansion of $\rho(E)$ in powers of the disorder strength, and by taking into account the nonperturbational effects of the disorder using the functional renormalization group. We have found that the average and the variance of the energy for one-dimensional interface of length L behave as, $\langle E\rangle_R \propto L \ln L$, $\Delta E_R \propto L$, while the distribution function of the energy tends for large L to the Gumbel distribution of the extreme value statistics.

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I. INTRODUCTION

The concept of energy landscapes is of current interest in different topics such as structural glasses, spin glasses, proteins, flux lines, etc. [1–8]. The existence of metastable states is crucial for the complex behavior in these systems. The domain wall counterpart of the random-field Ising model [9] provides an example of the problem which possesses complex properties, and can be quantitatively treated using the well established analytical methods such as the functional renormalization group (FRG) method [10] and the method of replica symmetry breaking (RSB) [11]. A significant progress has been achieved in recent years in understanding the behavior of interfaces in disordered media at equilibrium [10] and the driven interfaces at the depinning transition [12–14]. It is expected that in equilibrium or below the depinning transition there are many metastable states constituting the energy landscape. This makes the interface problem a natural candidate to study the concepts of energy landscapes. For recent theoretical and numerical studies of the related systems under the perspective of the energy landscape see Refs. [15–21]. In this paper we present the results of the study of the distribution function (DF) of the energy $\rho(E)$ for an interface in a random-field environment at equilibrium at zero temperature, i.e., in the ground state. The main result of this paper is that for large interfaces $\rho(E)$ is a universal function which coincides with the Gumbel distribution of the extreme value statistics. The dynamic formalism we use here can be applied to the study of the dynamic quantities such as the relaxation of the energy, the two times energy correlation functions, etc., where the complicated features of the energy landscape such as metastable states can be probed.

The paper is organized as follows. Section II introduces the model of elastic interfaces in a disordered medium. In Sec. III the energy DF for interfaces in a random field environment at zero temperature is obtained by summing the leading terms in the perturbation expansion. In Sec. IV the nonperturbational effects of the disorder are taken into account by using the FRG. Section V contains discussion of our results and elucidates their connection with the extreme value statistics. Final section contains our conclusions.

II. MODEL AND FORMALISM

The interface motion in a disordered medium at $T=0$ is described by the equation

$$\mu^{-1}\frac{\partial z(x,t)}{\partial t}=\gamma\nabla^2 z+F+g(x,z), \tag{1}$$

where μ is the mobility, γ is the stiffness constant, and F is the driving force density. The quenched random force $g(x,z)$ is assumed to be Gaussian distributed with the zero mean and the second cumulant $\langle g(x,z)g(x',z')\rangle=\delta^d(x-x')\Delta(z-z')$, where d is the interface dimension. To make this model well defined one has to introduce the cutoff Λ_0^{-1} in the $\delta^d(x)$ function at scales of order of the impurity separation or other microscopic scales. We restrict our consideration to the case of random-field disorder when the correlator $\Delta(z)=\Delta(-z)$ is a monotonically decreasing function of z for $z>0$ and decays rapidly to zero over a finite distance a.

It is well known that the Langevin equation (1) can be reformulated in terms of the Fokker-Planck equation for the conditional probability density $P(z(x),t;z^0(x),t^0)$ to have the profile $z(x)$ at time t by having the profile $z^0(x)$ at time t^0. This Fokker-Planck equation can be written as an integral equation, which, for an interface of a finite length L, reads

$$P(z,t;z^0,t^0)=P_0(z,t;z^0,t^0)-\mu\int_{t_0}^{t}dt'\int \mathcal{D}z' P_0(z,t;z',t') \times\sum_{k'}\partial_{z'_{k'}}g_{k'}(z')P(z',t';z^0,t^0), \tag{2}$$

where $z_k=\int d^dx z(x)\exp(-ikx)$ and $g_k(z)=\int d^dx \exp(-ikx)g(x,z)$, $[k=(k_1,\ldots,k_d),k_i=2\pi j_i/L,j_i=0,\pm 1,\ldots]$ are the Fourier transforms of the interface height and the quenched force, respectively. $\int\mathcal{D}z$ in Eq. (2) stays for integrations over the modes $z\equiv\{z_k\}$. The bare conditional probability reads

$$P_0(z,t;z^0,t^0)=\prod_k \delta[z_k-z_k^0\exp(-\gamma\mu k^2(t-t^0))] \times\delta[z_0-z_0^0-\mu F(t-t^0)]. \tag{3}$$

Analogous to the case of one Brownian particle [22] the formal solution of Eq. (2) averaged over disorder can be represented as a path integral

$$P_{\rm av}(z,t;z^0,t^0)=\int_{z(x,t^0)=z^0(x)}^{z(x,t)=z(x)}\mathcal{D}z\int \mathcal{D}p\exp(-S), \quad (4)$$

where the "action" $S=S_0+S_i$ is given by

$$S_0=-i\int_{t^0}^{t}dt'\int d^dx p(x,t')\times[\mu^{-1}\partial z(x,t')/\partial t'-\gamma\nabla^2 z(x,t')-F], \quad (5)$$

$$S_i=\frac{1}{2}\int_{t^0}^{t}dt'\int_{t^0}^{t}dt''\int d^dx p(x,t')\times\Delta[z(x,t')-z(x,t'')]p(x,t''), \quad (6)$$

with $p(x,t)$ being the momentum (response field) conjugated to the interface height $z(x,t)$. Notice that the correct mathematical definition of the path integral (4) is given through its discretized version.

III. ENERGY DISTRIBUTION

We now will consider the probability DF of the total energy of the elastic interface $E(z)=E_{\rm el}(z)+E_{\rm dis}(z)$, which can be split into the elastic energy $E_{\rm el}(z)=(\gamma/2)\int d^dx(\nabla z)^2$ and the disorder energy $E_{\rm dis}(z)=-\int d^dx\int_0^{z(x)}dz' g(x,z')$. The energy DF can be calculated using the conditional probability density $P(z(x),t;0,0)$ as follows:

$$\rho(E(t))=\int \mathcal{D}z(x)\,\delta(E-E(z))P(z(x),t;0,0). \quad (7)$$

The calculation of DF (7) requires in general summations of infinite series of Feynman diagrams, which can be classified by the number of loops. To the lowest order we take into account only one-loop diagrams, which contain one integration over an internal momentum. The loop expansion provides the bare expression for the energy DF, which will be further improved by using the renormalization group method.

It is convenient instead of $\rho(E(t))$ to consider its Fourier transform $\hat{\rho}(s)$ (characteristic function) which is obtained from Eq. (7) as

$$\hat{\rho}(s)=\sum_{n=0}^{\infty}\frac{(-is)^n}{n!}\langle E^n(t)\rangle =\sum_{n=0}^{\infty}\frac{(-is)^n}{n!}\sum_{m=0}^{n}C_n^m\langle E_{\rm el}^m(t)E_{\rm dis}^{n-m}(t)\rangle, \quad (8)$$

where $C_n^m=n!/(m!(n-m)!)$ denotes the binomial coefficient. In this paper we will restrict ourselves to the study of the energy DF in the steady state, i.e., for $t\to\infty$. In this limit $\langle E_{\rm el}^m E_{\rm dis}^{n-m}\rangle$ is related to the static equilibrium correlation function

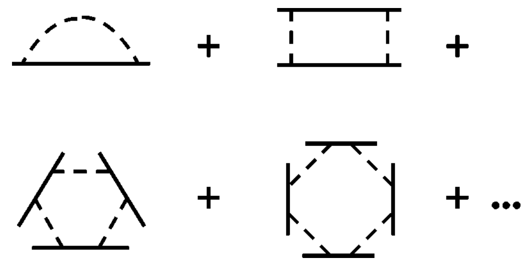

FIG. 1. The loop expansion of connected diagrams contributing to the energy distribution function.

$$\langle z(x_1)z(x_2)\rangle=\lim_{t\to\infty}\int \mathcal{D}z(x)z(x_1)z(x_2)P(z(x),t;0,0) =\int_k\frac{\Delta(0)}{(\gamma k^2)^2}e^{ik(x_1-x_2)}. \quad (9)$$

Let us first elucidate the computation of the nth moment of the elastic energy, which can be used to calculate the probability DF of the elastic energy. Expressing $E_{\rm el}$ through the Fourier components of the interface height $z(x)$ we obtain

$$\langle E_{\rm el}^n\rangle=\left(\frac{\gamma}{2}\right)^n\int \mathcal{D}z\int_{k_1}k_1^2|z_{k_1}|^2\cdots\int_{k_n}k_n^2|z_{k_n}|^2P(z,t;0,0). \quad (10)$$

For an interface of a finite size L the integral $\int_k$ means $L^{-d}\Sigma_k$. To compute Eq. (10) to the lowest order in disorder strength we iterate Eq. (2) $2n$ times and insert it into Eq. (10). Expecting that the steady state does not depend on the initial interface configuration we have taken the latter in Eqs. (9) and (10) to be flat at $t_0=0$. The average over the random forces, which is carried out by using the Wick theorem, yields connected and disconnected expressions. The connected expression contains only one integration over k, while the number of integrations over k in a disconnected expression is equal to the number of connected parts in that expression. Let us consider the calculation of the connected part of $\langle E_{\rm el}^n\rangle$. As a result of integrations by parts in Eq. (10) with $P(z,t;0,0)$ being iterated $2n$ times the $2n$ derivatives with respect to $z'_{k'_i}$ [see Eq. (2)] will act on z_{k_i} in Eq. (10). This has the consequence that pairs of $2n$ momenta $k'_1,\ldots,k'_{2n}$ associated with the right-hand side of Eq. (2) (being iterated) become consecutively equal to one of $k_1,\ldots,k_n$ in Eq. (10). There exist $(2n)!$ such possibilities. The factor $1/(2n)!$ results from getting rid of $2n$ ordered time integrations in $P(z,t;0,0)$. The number of possibilities to get a connected loop diagram is shown in Fig. 1 with n continuous lines is $2^{n-1}(n-1)!$. Integrations over $x_1,\ldots,x_{2n-1}$ arising from the above expression of $g_k(z)$ provides that the momenta of the modes being connected by a dashed line, which is associated with the disorder correlator, become equal. The integration over x_{2n} gives the factor L^d. The intermediate z'_k are zero for flat initial interface configuration due to δ functions in Eq. (3). As a result the arguments of disorder correlators

$\Delta(z)$ become zero. Collecting all combinatorial factors $1/(2n)!\cdot(2n)!\cdot 2^{n-1}(n-1)!=2^{n-1}(n-1)!$ and taking the limit $t\to\infty$ we find the following expression of the connected part of $\langle E_{\rm el}^n\rangle$

$$\frac{1}{n!}\langle E_{\rm el}^n\rangle_c=\frac{1}{2n}\Delta(0)^n\gamma^{-n}L^d\int_k\frac{1}{(k^2)^n}. \tag{11}$$

The moment of the elastic energy $\langle E_{\rm el}^n\rangle$ is expressed through the connected moments according to

$$\frac{1}{n!}\langle E_{\rm el}^n\rangle=\sum\frac{1}{m_1!\cdots m_N!}\left(\frac{\langle E_{\rm el}^{n_1}\rangle_c}{n_1!}\right)^{m_1}\cdots\left(\frac{\langle E_{\rm el}^{n_N}\rangle_c}{n_N!}\right)^{m_N},$$

where summations occur over $n_i>0$ and $m_i\geqslant 0$ fulfilling the condition $n_1m_1+\cdots+n_Nm_N=n$. As a consequence, the series of the DF of the elastic energy $\hat{\rho}_{\rm el}(s)$ over the disconnected moments is equal to the exponential of the sum over connected moments (connectivity theorem [23])

$$\sum_{n=0}^{\infty}\frac{(-is)^n}{n!}\langle E_{\rm el}^n\rangle=\exp\left(\sum_{n=1}^{\infty}\frac{(-is)^n}{n!}\langle E_{\rm el}^n\rangle_c\right). \tag{12}$$

The identity (12) can be considered as definition of the cumulants of the elastic energy. The use of Eq. (11) gives finally the distribution function of the elastic energy as

$$\hat{\rho}_{\rm el}(s)=\exp\left(-\frac{1}{2}L^d\int_k\ln\left(1+\frac{is\Delta(0)}{\gamma k^2}\right)\right). \tag{13}$$

The computation of $\langle E_{\rm el}^mE_{\rm dis}^{n-m}\rangle$ to the same order is similar to that of $\langle E_{\rm el}^n\rangle$. Expressing the random potential through the force according to $V(x,z(x))\equiv-\int_0^{z(x)}dz'g(x,z')$ we arrive at

$$\langle E_{\rm el}^mE_{\rm dis}^{n-m}\rangle=\left(\frac{\gamma}{2}\right)^m\int\mathcal{D}z\int_{k_1}k_1^2|z_{k_1}|^2\cdots\int_{k_m}k_m^2|z_{k_m}|^2$$
$$\times\int d^dx_{m+1}V(x_{m+1},z(x_{m+1}))\cdots$$
$$\times\int d^dx_nV(x_n,z(x_n))P(z,t;0,0). \tag{14}$$

To compute Eq. (14) to one-loop order we now iterate Eq. (2) $n+m$ times. As a result of integrations by parts in Eq. (14) the $2m$ derivatives with respect to $z'_{k'_i}$ will act on z_{k_i} while the rest n-m derivatives will act on $V(x_i,z(x_i))$. Similar to the case of the pure elastic energy, the pairs of $2m$ momenta taken from the momenta $k'_1,\ldots,k'_{m+n}$ associated with the iterated Eq. (2) become consecutively equal to one of $k_1,\ldots,k_m$ in Eq. (14). There exists now $(n+m)!/(n-m)!$ such possibilities. The factor $1/(n+m)!$ results from getting rid of $n+m$ ordered time integrations in $P(z,t;0,0)$. Only terms with the first order derivatives of V with respect to $z'_{k'_i}$ survive after averaging over disorder in the one-loop approximation, so that the factor $(n-m)!$ results from the differentiation of V. Using the relation $\int d^dx\partial_{z_k}V(x,z(x))=-L^{-d}g_{-k}(z)$ we express all $V(x,z(x))$ through $g_{-k_i}(z)$ with $-k_i$ being equal to one of the free n-m momenta associated with the iterated Eq. (2). The following calculation is identical to that for $\langle E_{\rm el}^n\rangle$. Averaging over disorder and collecting all combinatorial factors we obtain the connected part in the form

$$\langle E_{\rm el}^mE_{\rm dis}^{n-m}\rangle_c=\frac{1}{2}(-2)^{n-m}(n-1)!\Delta(0)^n\gamma^{-n}L^d\int_k\frac{1}{(k^2)^n}. \tag{15}$$

The use of Eq. (15) in Eq. (8) yields

$$\frac{1}{n!}\langle E^n\rangle_c=\frac{1}{n!}\sum_{m=0}^{n}C_n^m\langle E_{\rm el}^m(t)E_{\rm dis}^{n-m}(t)\rangle_c$$
$$=\frac{(-1)^n}{2n}\Delta(0)^n\gamma^{-n}L^d\int_k\frac{1}{(k^2)^n}, \tag{16}$$

which can be obtained from Eq. (11) multiplying it with the factor $(-1)^n$. Thus, the expression $1/n!\langle E^n\rangle_c$ is associated with the loop diagram consisting of n continuous lines (see Fig. 1). The factor $2n$ in Eq. (16) is the symmetry number of the corresponding diagram. The straightforward analysis gives that the expansion (8) can be represented as a series of loop diagrams. The use of the connectivity theorem (12) enables us to write the Fourier transform of the energy DF (8) as exponential of the series of connected loop diagrams shown in Fig. 1

$$\hat{\rho}(s)=\exp\left(-\frac{1}{2}L^d\int_k\ln\left(1-\frac{is\Delta(0)}{\gamma k^2}\right)\right). \tag{17}$$

Note that $\hat{\rho}(s)$ given by the diagram series in Fig. 1 is closely related to the loop expansion of the effective potential in quantum field theory studied in Ref. [24]. Replacing the integral in Eq. (17) by the sum according to $L\int_kf(k)\to\sum_{j=-\infty}^{\infty}f(2\pi j/L)$ we find in $d=1$

$$\hat{\rho}(s)=\prod_{j=1}^{\infty}(1+isE_0/j^2)^{-1}=\frac{\pi\sqrt{isE_0}}{\sinh(\pi\sqrt{isE_0})}, \tag{18}$$

where $E_0=-\Delta(0)L^2/(4\pi^2\gamma)$ is the characteristic energy for an interface with the perturbational roughness $w\propto L^{3/2}$, which follows from $w\propto L^{(4-d)/2}$ for $d=1$.

Equation (18) has only simple poles $s=ij^2/E_0$ in the lower half plane, so that the inverse Fourier transformation of Eq. (18) can be easily performed as a sum over all poles by using Jordan's lemma. As a result we obtain the DF as $\rho(E)=|E_0|^{-1}f(E/E_0)$, $E<0$, where

$$f(x)=2\sum_{j=1}^{\infty}(-1)^{(j+1)}j^2e^{-xj^2}. \tag{19}$$

The comparison of Eqs. (11) and (16) shows that Eq. (19) describes also the DF of the elastic energy ($x=E/|E_0|>0$) [see also (13) and (17)] and the disorder energy ($x=E/2E_0>0$). This represents a generalization of the virial theorem for average energies to the corresponding probability DFs. Equation (19) coincides with the dimensionless width DF for the one-dimensional random-walk interface studied in Ref. [25]. Using the method of stationary phase it was shown in Ref. [25] that function (19) can be well approximated for small x by $f(x)\approx\sqrt{\pi/x^5}(\pi^2/2-x)e^{-\pi^2/4x}$. Using Eq. (19) we have computed the average energy, $\langle E\rangle=\pi^2E_0/6$, and the variance $\Delta E=(\langle E^2\rangle-\langle E\rangle^2)^{1/2}=\pi^2|E_0|/(3\sqrt{10})$. Note that Eq. (19) for the DF of elastic energy is the exact perturbational result generalizing the result established by Efetov and Larkin [26] for the height-height correlation functions (9), which can be readily proved by using supersymmetry [27]. Contrary to this, Eq. (19) for the DF of total or disorder energy has only been proved to one-loop order. Nevertheless, both Eqs. (9) and (19) are wrong due to the fact that Eq. (9) gives the value $(4-d)/2$ for the roughness exponent instead of the correct Imry-Ma [28] value $\zeta=(4-d)/3$.

IV. RENORMALIZATION

We now will take into account the effect of the renormalization on the energy DF using the results of the FRG [10]. After integrating out fluctuations in the functional (4) in the momentum shell $l^{-1}<k<\Lambda_0$, where l^{-1} is the new upper cutoff, we obtain renormalized quantities which depend on scale l. The corresponding flow equation for the renormalized correlator reads [10,12]

$$\frac{d\Delta(z)}{d\ln l}=-\frac{l^{\varepsilon}}{8\pi^2\gamma^2}\frac{d^2}{dz^2}\left[\frac{1}{2}\Delta^2(z)-\Delta(z)\Delta(0)\right], \quad (20)$$

where $\varepsilon=4-d$, so that $d=4$ is the upper critical dimension. The flow takes the correlator $\Delta(z)$ through a special point corresponding to the Larkin scale $L_c\simeq[\gamma^2a^2/\Delta(0)]^{1/\varepsilon}$, where it acquires a cusp at the origin $z=0$. Beyond the Larkin scale the renormalized correlator becomes singular and the perturbation theory breaks down. Nevertheless, the flow tends to the nontrivial fixed-point solution

$$\Delta(l,z)=8\pi^2\gamma^2A^{2/3}l^{2\zeta-\varepsilon}\Delta^*(zA^{-1/3}l^{-\zeta}), \quad (21)$$

which controls the large scale behavior. To determine the exponent ζ one has to consider the integral $I_\Delta=\int_{-\infty}^{\infty}dz\Delta(l,z)$, which is an invariant of the flow equation (20), and $I^*=\int_{-\infty}^{\infty}dz\Delta^*(z)$. The random field case corresponds to the fixed point characterized by $I_\Delta>0$ and $I^*=I_\Delta l^{\varepsilon-3\zeta}/(8\pi^2\gamma^2A)=\text{const}$ [10,12], so that $\zeta=\varepsilon/3$. According to Eq. (21) the renormalized disorder correlator $\Delta_R(0)$ acquires in the vicinity of the fixed point the scale dependence $l^{2\zeta-\varepsilon}$. Taking into account the latter in Eq. (9) by making the substitution $\Delta(0)\to\Delta(0)_R=\Delta(0)[k/k_c]^{\varepsilon-2\zeta}$ results in $\langle z(x_1)z(x_2)\rangle\propto|x_1-x_2|^{2\zeta}$, and therefore, gives the correct value of the roughness exponent ζ.

To enable a crossover to the perturbational regime at small length scales $l<L_c$ we use the ansatz

$$\Delta_R(0)=\Delta(0)[1+(k/k_c)^{2\zeta-\varepsilon}]^{-1}, \quad (22)$$

where the wave vector $k_c=2\pi/L_c$ is associated with the Larkin length L_c. The ansatz (22) describes the scale dependence of $\Delta_R(0)$ at the cusped fixed-point solution of the disorder correlator, $\Delta_R(0)\simeq\Delta(0)[k/k_c]^{\varepsilon-2\zeta}$ for $k\ll k_c$, and describes the crossover to the perturbational regime, $\Delta_R(0)\simeq\Delta(0)$ for $k\gg k_c$. Using the renormalized $\Delta_R(0)$ in Eq. (17) we obtain the Fourier transform of the renormalized distribution of the energy in $d=1$ as

$$\hat{\rho}_R(s)=\prod_{j=1}^{\infty}\left(1+\frac{is\tilde{E}_0}{j(1+j/\eta)}\right)^{-1}, \quad (23)$$

where $\eta=L/L_c$ and $\tilde{E}_0=-\Delta(0)L_cL/(4\pi^2\gamma)$. Similar to Eq. (18) the Fourier transform (23) has only simple poles $s=ij(1+j/\eta)/\tilde{E}_0$ in the lower half plane. Carrying out the inverse Fourier transformation of Eq. (23) by summing over all poles we obtain $\rho_R(E)=|\tilde{E}_0|^{-1}f_R(E/\tilde{E}_0;\eta)$, where the function $f_R(x;\eta)$ is given by

$$f_R(x;\eta)=\sum_{j=1}^{\infty}(-1)^{j+1}\frac{\Gamma(j+\eta+1)(1+2j/\eta)}{\Gamma(\eta+1)\Gamma(j)}e^{-j(1+j/\eta)x}. \quad (24)$$

For string lengths much shorter than the Larkin length, $\eta\ll1$, the DF (24) passes over to the perturbational result (19). Similar to the height-height correlation function at equilibrium we expect that Eq. (24), which is the result of the renormalization of Eq. (19) to order ε is exact. Equation (24) applies to order ε at the depinning transition too with the difference that in this case there are corrections to (24) of order ε^2. However, we expect that the latter will be small as it is the case for corrections of order ε^2 to the interface width distribution at the depinning transition [29]. The average energy $\langle E\rangle_R$ derived from Eq. (23) is

$$\begin{aligned}\langle E\rangle_R&=\tilde{E}_0\sum_{j=1}^{\infty}\frac{1}{j(1+j/\eta)}\\&=[\Psi(\eta+1)+C]\tilde{E}_0\\&\simeq[\ln\eta+C]\tilde{E}_0+O(1/\eta)\propto L\ln L,\end{aligned} \quad (25)$$

where $\Psi(x)$ is the digamma function and $C=0.5772\ldots$ is Euler's constant. The calculation of $\langle E\rangle_R$ with the use of DF (24) leads to an alternating series, the equivalence of which to Eq. (25) has been checked numerically. The energy fluctuation ΔE_R obtained from Eq. (23) reads

$$\begin{aligned}\Delta E_R&=|\tilde{E}_0|\left[\frac{\pi^2}{6}+\Psi'(\eta+1)-2(C+\Psi(\eta+1))/\eta\right]^{1/2}\\&\simeq\frac{\pi}{\sqrt{6}}|\tilde{E}_0|+O(\ln\eta/\eta)\propto L.\end{aligned} \quad (26)$$

The result $\Delta E_R\propto L$ agrees with the estimate of the energy by using dimensionality arguments with correct roughness ex-

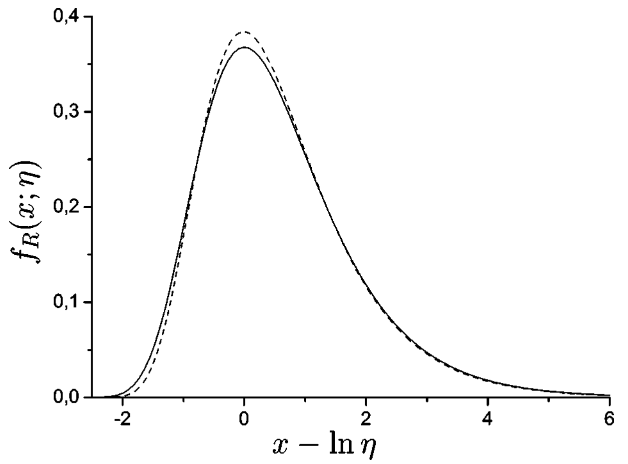

FIG. 2. The renormalized distribution of the energy for a line in a random-field environment. Dashed line: $L/L_c = 10^2$; solid line: the Gumbel distribution.

ponent ζ. Note that due to the logarithmic term in Eq. (25) $\langle E\rangle_R$ and ΔE_R scale in different way, and the relative fluctuation $\Delta E_R/\langle E\rangle_R$ disappears as $1/\ln L$ for large L, which is in contrast to $1/\sqrt{L}$ behavior for a Gaussian distribution. The latter reflects the relevance of fluctuations over all length scales. The higher moments of the energy distribution (24) scale as $\langle(E-\langle E\rangle)^n\rangle \propto \Delta E_R^n$. Figure 2 shows $f_R(x;\eta)$, which is given by Eq. (24), as a function of $x - \ln\eta$.

We now will consider the asymptotic behavior of (24) in the limit of long lines, $L \gg L_c$. Changing x in favor of $x - \ln\eta \equiv y$ and taking the limit $\eta\to\infty$ we calculate the sum over j in Eq. (24) and arrive at

$$f_R(y) = \mathcal{P}(y) \equiv \exp(-y - \exp(-y)), \tag{27}$$

which is nothing but the Gumbel distribution of the extreme value statistics [30]. The universality of $f_R(y)$ is due to the universal character of fluctuations on large scales, which are described by the fixed-point solution of the FRG [10]. Note that the expectation value of y calculated with Eq. (27) gives Euler's constant C which is in consistence with Eq. (25) of the average energy. We have checked that the limit of the distribution $f_R(x;\eta)$ for $\eta\to\infty$ is insensitive to the details of the renormalization at scales smaller than the Larkin scale.

The Gumbel distribution is one of the three possible limit distributions in the extreme value statistics [30], which concerns the distribution of the maximum $M_n = \max\{\xi_1, \ldots, \xi_n\}$ (or minimum) of the set of identically distributed random variables ξ_i $(i=1,2,\ldots,n)$. The asymptotic distribution $P_n(x)$ for M_n in limit $n\to\infty$ does not depend on details of the distribution of ξ_i and under fulfilling some conditions [30] has the form $P_n(x) \simeq \mathcal{P}(x - \ln n)$ where $\mathcal{P}(y)$ is given by Eq. (27) [for minimum $\mathcal{P}(-y)$]. The combination $y = x - \ln n$, where n is the number of random variables guarantees that the distribution remains invariant for $n\to\infty$.

Vinokur *et al.* [16] have used the Gumbel distribution to describe in a phenomenological way the energy barriers distribution for a flux line in a random environment. The creep motion of the flux line in the limit of small driving force F and low temperature is controlled by thermally activated jumps. The thermally activated advance of the flux segment of length L is controlled by the global barrier $U = \max\{U_1, \ldots, U_n\}$, where U_i is the barrier for the subsegment i of length L_c with the number of subsegments $n = L/L_c$. It was suggested in [16] that the probability distribution for a given segment L is $\mathcal{P}(U/U_c - \ln(L/L_c))$, where $U_c \simeq \gamma a^2 L_c^{d-2}$ is the minimum average barrier between neighboring metastable positions of a pinned segment L_c, so that the typical barrier of a segment of length L scales then as $U \propto U_c \ln(L/L_c)$. Bouchaud and Mézard [19] showed that the Gumbel distribution describes the energy distribution in a class of random energy models possessing the one-step RSB. The Gumbel and related distributions were used in Ref. [31], to describe universal fluctuations in correlated systems. It was shown in Ref. [32] that the Gumbel distribution appears in systems with $1/f$ power spectra. The Gumbel distribution applies also in the theory of statistical significance in protein and DNA sequence analysis [33].

V. DISCUSSION

We now will discuss the difference between the perturbational result for the energy DF (19) and that obtained using the results of FRG (24). Further, we will also attempt to interpret the asymptotic behavior of the energy DF (24), which in the limit of long lines coincides with the Gumbel distribution (27), in terms of extreme value statistics. It is known from the treatment of the problem in the framework of the replica variational approach [11] that the system under consideration demonstrates RSB. The RSB consideration [11] gives the same height-height correlation function and the same roughness exponent ζ as the ones predicted by the FRG, and consequently the same energy DF. The RSB is related to the existence of multiple minima of corresponding Hamiltonian at $T=0$. The failure of the perturbation theory due to the existence of multiple minima was clarified in toy models of a domain wall in a random field Ising model [34,35]. According to Ref. [34] the perturbational result (19) for the energy DF at $T=0$ can be interpreted as the DF of an average over all multiple minima (and even maxima) of the Hamiltonian with the same weights. The latter means that all multiple minima contribute to Eq. (19) independent of their depths resulting in $w \propto L^{3/2}$ and $\langle\Delta E\rangle \propto L^2$, which are in contrast to the results $w \propto L$ and $\langle\Delta E\rangle \propto L$ which are expected to be exact. The renormalization changes the weights for different minima, so that the deeper minima are taken into account with larger weights. This is in agreement with the result that the renormalized average energy $\langle E\rangle_R \propto L\ln L$ becomes lower than that predicted by perturbation theory, and that the energy fluctuation $\langle\Delta E\rangle_R \propto L$ becomes smaller. The effect of the renormalization depends on $\eta = L/L_c$, so that the contribution of the lower minima to Eq. (27) will become more pronounced in the limit of a long line ($\eta\to\infty$). Consequently, the fact that the interface will preferentially occupy the lowest energy state, which is the minimum of many random variables being the local energy minima, is taken into account in the renormalized DF. Therefore, the energy DF is expected to be related to extreme value statistics. The present work explicitly shows that the distribution of the energy of a line in a random-field environment is given by the Gumbel distribution.

VI. CONCLUSIONS

We have studied the distribution function of the elastic, disorder, and the total energies of an interface in a random-field environment at zero temperature by summing the leading terms of the perturbation expansion in powers of the disorder. The nonperturbational effects of the disorder are taken into account using the FRG method. We have found that the average and the fluctuation of the energy for one-dimensional interfaces behave as $\langle E\rangle_R \propto L \ln L$, $\Delta E_R \propto L$, while the energy DF tends for large L to a universal function which coincides with the Gumbel distribution of extreme value statistics. The more complicated features of the energy landscape are expected to be probed in considering the dynamic quantities such as the two times energy correlation functions, etc., which can be studied by using the present method.

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